Modification rules for orthosymplectic superalgebras. II. Atypical representations

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LETTER TO THE EDITOR

Modification rules for orthosymplectic superalgebras: II. Atypical representations

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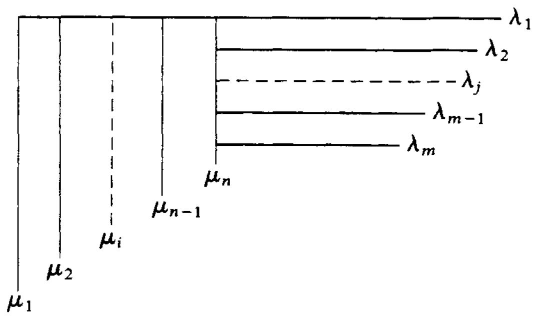
Abstract. Modification rules are presented for all finite-dimensional, atypical representations of OSp(M/N). With the representations specified by Young supertableaux, these rules relate non-standard supertableaux to standard supertableaux via the removal of continuous boundary hooks.

In a recent paper, Farmer (1986) (hereafter referred to as I) presented modification rules for finite-dimensional graded tensor representations of OSp(M/N). These rules covered all typical tensor representations together with atypical representations which satisfied up to two atypicality conditions. In this letter, these rules are extended to provide modification rules for *all* finite-dimensional tensor representations, both typical and atypical. In the interests of succinctness, the results of I will not be repeated here and the reader is advised to read the present work in conjunction with I.

For OSp($2m+1/2n$) and OSp($2m/2n$) the tensor representations are specified here by Young supertableaux (Farmer and Jarvis 1984). A *standard* Young supertableau is of the form:

$[\lambda] =$



(1)

where λ_j is the number of boxes beyond the nth column in the jth row, with $j \leqslant m$, and μ_i is the number of boxes in the ith column, with $i \leqslant n$. *Non-standard* supertableaux include boxes outside the standard $(m \times n)$ envelope of (1). The modification rules presented here relate non-standard supertableaux to standard supertableaux via the removal of continuous boundary hooks.

As discussed in I, non-standard supertableaux are called atypical if they satisfy any of the conditions:

$$\mu_i + n + j + 1 = \lambda_j + M + i \tag{2}$$

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where $1 \leq i \leq n$, $1 \leq j \leq m$, and $M = 2m+1$ for OSp$(2m+1/2n)$ or $M = 2m$ for OSp$(2m/2n)$. We denote an atypicality condition (2) which relates μ_i and λ_j by (i, j) and impose an ordering $(i, j) > (k, l)$ for $i > k$ and $j > l$. The maximum number of atypicality conditions which may be simultaneously realised is the lesser of m and n.

The modification rules for a non-standard supertableau $[\lambda]$, which satisfies simultaneously A atypicality conditions, (i_k, j_k), $k = 1, \ldots, A$, where $(i_k, j_k) < (i_{k+1}, j_{k+1})$, are

$$\begin{aligned}
[\lambda] = [\lambda^\mu] &- \sum_{k_1=1}^{A} P_{k_1}([\lambda - h_{k_1}] - [\lambda^\mu - h_{k_1}]) \\
&- \sum_{\substack{k_1,k_2 \\ k_1<k_2}}^{A} P_{k_2}P_{k_1}([\lambda - h_{k_2} - h_{k_1}] - [\lambda^\mu - h_{k_2} - h_{k_1}]) \\
&- \sum_{\substack{k_1,k_2,k_3 \\ k_1<k_2<k_3}}^{A} P_{k_3}P_{k_2}P_{k_1}([\lambda - h_{k_3} - h_{k_2} - h_{k_1}] - [\lambda^\mu - h_{k_3} - h_{k_2} - h_{k_1}]) - \ldots \\
&- \sum_{\substack{k_1,k_2,\ldots,k_A \\ k_1<k_2<\ldots<k_A}}^{A} P_{k_A}P_{k_{A-1}} \ldots P_{k_1}([\lambda - h_{k_A} - h_{k_{A-1}} - \ldots - h_{k_1}] \\
&- [\lambda^\mu - h_{k_A} - h_{k_{A-1}} - \ldots - h_{k_1}]),
\end{aligned} \tag{3}$$

where (λ^μ) is obtained from (λ) by applying the modification rules for typical supertableaux which are presented in I. The h_k are continuous boundary strips to be removed from (λ) or (λ^μ) starting from the end box in column i_k and finishing at the end box in row j_k. It is essential to perform these hook removals in the order shown in (3) reading from left to right. Finally, the P_k are phase factors given by

$$P_k = (-1)^{\lambda_j - i_k - n - 1}. \tag{4}$$

As an example of (3), we show how a non-standard supertableau, which satisfies simultaneously four atypicality conditions, modifies in OSp(8/8). The supertableaux will be denoted simply by row lengths $[r_1, r_2, \ldots]$. If we take $[\lambda] = [6, 6, 6, 6, 6]$ then with reference to (1) we have

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 2 \qquad \text{and} \qquad \mu_1 = \mu_2 = \mu_3 = \mu_4 = 5.$$

Thus from (2) we see that four atypicality conditions are satisfied, i.e. $\mu_i = \lambda_i + 3$ for $i = 1, 2, 3$ and 4. From I we find that $[\lambda^\mu] = -[6, 6, 6, 6, 4]$. Thus from (3) we find $[\lambda]$ modifies as follows:

$$\begin{aligned}
[6,6,6,6,6] = &-[6,6,6,6,4] - [5,5,5,5] - [5,5,5,3] + [6,5,5,5,1] + [6,5,5,3,1] \\
&- [6,6,5,5,2] - [6,6,5,3,2] + [6,6,6,5,3] + [6,6,6,3,3] + [4,4,4] \\
&+ [4,4,2] - [5,4,4,1] - [5,4,2,1] + [6,4,4,1,1] \\
&+ [6,4,2,1,1] + [6,6,4,2,2] + [6,6,2,2,2] - [6,5,4,2,1] \\
&- [6,5,2,2,1] + [5,5,4,2] + [5,5,2,2] + [3,3] + [3,1] + [5,3,1,1] \\
&+ [5,1,1,1] - [4,3,1] - [4,1,1] - [6,3,1,1,1] - [6,1,1,1,1] - [2] - [0].
\end{aligned}$$

These rules have been deduced by decomposing partitions $[\lambda]$, of $\mathrm{OSp}(M/N)$, according to the branching rule

$$\mathrm{OSp}(M/N)\downarrow \mathrm{O}(M)\times \mathrm{Sp}(N) \qquad [\lambda]\downarrow \sum_{\zeta}[\zeta/D]\langle\widetilde{\lambda/\zeta}\rangle \tag{5}$$

where ζ is any partition, the D series consists of all partitions with even row lengths and modifications can be carried out using the known results for $\mathrm{O}(M)$ and $\mathrm{Sp}(N)$ (King 1971). With the aid of the group theory computer package SCHUR these results have been checked up to partitions of rank 25 in OSp(9/8), which is the smallest partition satisfying four atypicality conditions simultaneously.

Further checks on the consistency of these results have been made by the following procedure:

(*a*) a non-standard supertableau, $[\lambda]$, in $\mathrm{Osp}(M/N)$ is modified using (3) and the resulting expression is decomposed according to the branching rule

$$\mathrm{OSp}(M/N)\downarrow \mathrm{OSp}(M-1/N) \qquad [\lambda]\downarrow[\lambda/M] \tag{6}$$

or

$$\mathrm{OSp}(M/N)\downarrow \mathrm{OSp}(M/N-2) \qquad [\lambda]\downarrow[\lambda/MM], \tag{7}$$

where the M series consists of all partitions with only one part. Further modifications are then performed, where necessary, in $\mathrm{OSp}(M-1/N)$ or $\mathrm{OSp}(M/N-2)$;

(*b*) the non-standard $\mathrm{OSp}(M/N)$ supertableau $[\lambda]$ is decomposed, without modification, according to (6) or (7). Modifications in $\mathrm{OSp}(M-1/N)$ or $\mathrm{OSp}(M/N-2)$ are then performed where necessary. If the modification rules are internally consistent procedures (*a*) and (*b*) should result in identical expressions. Using this procedure the modification rules have been checked for partitions up to rank 49 in OSp(13/12), which is the smallest partition satisfying six atypicality conditions simultaneously.

These checks lend significant confidence to the results presented here; however, general proofs are still required and work continues to educe such proofs.

To conclude, some of the uses to which these results can be put are mentioned. In the evaluation of Kronecker products in $\mathrm{OSp}(M/N)$ (King 1983)

$$[\lambda]\times[\mu]=\sum_{\zeta}[(\lambda/\zeta)\cdot(\mu/\zeta)] \tag{8}$$

non-standard supertableaux may arise which require modification. They are also necessary for a number of branching rules such as (6) and (7) and also

$$\mathrm{SU}(M/N)\downarrow \mathrm{OSp}(M/N) \qquad \{\lambda\}\downarrow[\lambda/D] \tag{9}$$

and

$$\mathrm{OSp}(M_1+M_2/N_1+N_2)\downarrow \mathrm{OSp}(M_1/N_1)\times \mathrm{OSp}(M_2/N_2) \qquad [\lambda]\downarrow\sum_{\zeta}[\lambda/\zeta][\zeta/D]. \tag{10}$$

These results have now been incorporated into SCHUR through the efforts of M Hirst.

References

Farmer R J 1986 *J. Phys. A: Math. Gen.* **19** 321

Farmer R J and Jarvis P D 1984 *J. Phys. A: Math. Gen.* **17** 2365

King R C 1971 *J. Math. Phys.* **12** 1588

—— 1983 *Lecture Notes in Physics* **180** 41 (Berlin: Springer)